A Bernstein-density proof of Erdős's robust interpolation obstruction

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Abstract

We prove the robust interpolation obstruction recorded as Erdős Problem #1133. Given $C > 0$, we show that there is $\varepsilon > 0$ such that, for all sufficiently large n and for every choice of nodes $x_1, \dots, x_n \in [-1, 1]$, one can assign labels $y_1, \dots, y_n \in [-1, 1]$ with the following property: every polynomial of degree $< (1 + \varepsilon)n$ that fits at least $(1 - \varepsilon)n$ of the prescribed pairs has uniform norm $> C$ on $[-1, 1]$.

The proof uses one external theorem: Beurling's interpolation theorem for Bernstein spaces, in the real-line form later extended by Ortega-Cerdà and Seip. The strict density bound for interpolation sequences in the Bernstein space $\mathcal{B}_1$ implies a finite obstruction: for each fixed norm bound C there are L and $\eta > 0$ such that every L-point multiset in an interval of length at most $\pi(1 + \eta)L$ carries a label pattern in $[-1, 1]$ which is not interpolable by any $f \in \mathcal{B}_1$ of norm at most C. This finite statement is derived here by a compactness and stationary-density argument.

Arbitrary algebraic nodes are then moved to angular coordinates $x = \cos\theta$ and partitioned into consecutive blocks of size L. Since the total angular length is π, more than εn blocks have angular span $O(L/n)$ when ε is chosen small enough. On each such block we place one of the finite forbidden Bernstein label patterns. A bounded algebraic polynomial fitting an entire good block rescales to a bounded element of $\mathcal{B}_1$ fitting the forbidden local data, a contradiction. Hence every admissible polynomial misses at least one point in each good block, and therefore misses more than εn labels.

1 Introduction

Erdős Problem #1133 asks for a robust obstruction to near-full interpolation by bounded low-degree polynomials [2]; the surrounding interpolation questions go back to Erdős's 1968 survey on Lagrange interpolation and extremal polynomial problems [3]. In the notation used below, the problem is the following.

Theorem 1.1 (Main theorem)**.** *Let $C > 0$. There exist $\varepsilon > 0$ and $n_0 \in \mathbb{N}$ such that, whenever $n \geq n_0$ and*

$$x_1, \dots, x_n \in [-1, 1]$$

are arbitrary nodes, counted with multiplicity, there are labels

$$y_1, \dots, y_n \in [-1, 1]$$

such that the following holds. If P is a real or complex polynomial with

$$\deg P < (1 + \varepsilon)n$$

and

$$P(x_i) = y_i$$

for at least $(1-\varepsilon)n$ *indices* i*, then*

$$\|P\|_{L^\infty[-1,1]} > C.$$

The proof is deterministic once the node set is given. Its key point is that the relevant local model for algebraic polynomials is not ordinary Lagrange interpolation at fixed degree, but interpolation in the Bernstein space after angular rescaling.

Put

$$x = \cos\theta.$$

If P has degree m, then

$$Q(\theta) := P(\cos\theta)$$

is an even trigonometric polynomial of degree at most m. Locally at scale $1/m$ in the θ-variable, trigonometric polynomials of degree m resemble functions of exponential type at most one. The global interpolation obstruction is therefore obtained by planting many local finite obstructions supplied by the strict Beurling density theorem for Bernstein spaces.

Remark 1.2 (Repeated nodes). The theorem is stated for multisets. If two equal nodes receive distinct labels, a polynomial cannot fit both copies. The proof below allows this and uses the same bookkeeping for ordinary and repeated nodes.

2 Bernstein spaces and interpolation density

For $\tau > 0$, let $\mathcal{B}_\tau$ denote the Bernstein space

$$\mathcal{B}_\tau := \{f \text{ entire} : \operatorname{type}(f) \le \tau,\ \|f\|_{\infty,\mathbb{R}} < \infty\},$$

where

$$\|f\|_{\infty,\mathbb{R}} := \sup_{x\in\mathbb{R}} |f(x)|.$$

Here $\operatorname{type}(f) \le \tau$ means that, for every $\delta > 0$, one has

$$|f(z)| \le A_\delta e^{(\tau+\delta)|z|}$$

for some $A_\delta < \infty$. For functions bounded on the real line this is equivalent, by the Phragmén–Lindelöf principle, to the more precise strip estimate

$$|f(x+iy)| \le \|f\|_{\infty,\mathbb{R}} e^{\tau|y|}, \qquad x, y \in \mathbb{R}. \tag{1}$$

We use the classical Bernstein inequality

$$\|f'\|_{\infty,\mathbb{R}} \le \tau \|f\|_{\infty,\mathbb{R}}, \qquad f \in \mathcal{B}_\tau. \tag{2}$$

A real sequence $\Lambda \subset \mathbb{R}$ is called separated if

$$\inf\{|\lambda - \mu| : \lambda, \mu \in \Lambda, \lambda \ne \mu\} > 0.$$

Its upper uniform Beurling density is

$$D^+(\Lambda) := \limsup_{R\to\infty} \sup_{a\in\mathbb{R}} \frac{\#(\Lambda \cap [a, a+R])}{R}.$$

Definition 2.1. A separated real sequence Λ is an interpolation sequence for $\mathcal{B}_\tau$ if for every bounded complex sequence $(a_\lambda)_{\lambda\in\Lambda}$ there exists $f \in \mathcal{B}_\tau$ such that

$$f(\lambda) = a_\lambda \qquad (\lambda \in \Lambda).$$

The only external theorem used in the proof is the following real-line case of Beurling's interpolation theorem.

Theorem 2.2 (Beurling interpolation theorem)**.** *Let $\tau > 0$, and let $\Lambda \subset \mathbb{R}$ be separated. Then Λ is an interpolation sequence for $\mathcal{B}_\tau$ if and only if*

$$D^+(\Lambda) < \frac{\tau}{\pi}.$$

In particular, if Λ is an interpolation sequence for $\mathcal{B}_1$, then

$$D^+(\Lambda) < \frac{1}{\pi}.$$

This is Beurling's theorem on the real line [1]. Ortega-Cerdà and Seip extended the result to complex interpolation sequences for Bernstein spaces, where an additional Carleson-type separation condition appears; for real sequences this condition reduces to ordinary separation and their density agrees with the usual upper uniform density displayed above [5, Theorem 1].

3 A finite obstruction in the Bernstein space

The next proposition is the main compactness consequence of Theorem 2.2. It is the only place where the strict density theorem is used.

Proposition 3.1 (Finite Bernstein obstruction)**.** *For every $C > 0$ there exist $\eta > 0$ and an integer $L \geq 1$ such that the following holds. Let*

$$u_1, \ldots, u_L \in \mathbb{R}$$

be any multiset with

$$\operatorname{diam}\{u_1, \ldots, u_L\} \leq \pi(1+\eta)L.$$

Then there exist real labels

$$a_1, \ldots, a_L \in [-1, 1]$$

such that no complex-valued $f \in \mathcal{B}_1$ satisfying

$$\|f\|_{\infty,\mathbb{R}} \leq C$$

has

$$f(u_j) = a_j, \qquad j = 1, \ldots, L.$$

We prove the proposition by contradiction. If it failed for a fixed C, then there would be arbitrarily long finite sets of almost critical average density on which every $[-1, 1]$ datum could be interpolated with the same norm bound C. Translating and averaging those finite sets gives a stationary point process of intensity at least $1/\pi$. Local compactness and Montel's theorem transfer the uniform finite interpolation property to configurations in the support of this process. An ergodic component then supplies a separated infinite interpolation sequence for $\mathcal{B}_1$ with upper density at least $1/\pi$, contradicting Theorem 2.2.

3.1 Configuration-space preliminaries

Fix $\delta > 0$. Let $\mathcal{X}_\delta$ be the space of all closed δ-separated subsets of $\mathbb{R}$, including the empty set. We identify a set $\Lambda \in \mathcal{X}_\delta$ with its counting measure

$$\mu_\Lambda := \sum_{\lambda \in \Lambda} \delta_\lambda.$$

The topology on $\mathcal{X}_\delta$ is the vague topology of these counting measures: $\Lambda_j \to \Lambda$ if and only if

$$\int \varphi \, d\mu_{\Lambda_j} \to \int \varphi \, d\mu_\Lambda \qquad \text{for every } \varphi \in C_c(\mathbb{R}).$$

Because every interval of length R contains at most $1 + R/\delta$ points of a δ-separated set, this space is compact and metrizable. The translation action is denoted by

$$T_t\Lambda := \Lambda - t := \{\lambda - t : \lambda \in \Lambda\}.$$

For $\varphi \in C_c(\mathbb{R})$ put

$$N_\varphi(\Lambda) := \int \varphi \, d\mu_\Lambda = \sum_{\lambda \in \Lambda} \varphi(\lambda).$$

By definition of the topology, N_φ is continuous on $\mathcal{X}_\delta$.

Lemma 3.2 (Stationary extraction)**.** *Let $C \geq 1$. Suppose there are sequences $L_k \to \infty$, $\eta_k \downarrow 0$, and finite sets*

$$U_k \subset [0, T_k], \qquad \#U_k = L_k, \qquad T_k \leq \pi(1 + \eta_k)L_k,$$

with the following finite interpolation property: for every assignment $a : U_k \to [-1, 1]$ there exists $f \in \mathcal{B}_1$ with

$$\|f\|_{\infty,\mathbb{R}} \leq C, \qquad f(u) = a(u) \quad (u \in U_k).$$

Then there exists a separated infinite sequence $\Lambda \subset \mathbb{R}$ which is an interpolation sequence for $\mathcal{B}_1$ and satisfies

$$D^+(\Lambda) \geq \frac{1}{\pi}.$$

Proof. First, the sets U_k are uniformly separated. If $u, v \in U_k$ are distinct, prescribe the two values 1 and -1 at u and v, and extend the data arbitrarily to the rest of U_k. The resulting interpolating function satisfies, by (2),

$$2 \leq |f(u) - f(v)| \leq \|f'\|_{\infty,\mathbb{R}} |u - v| \leq C|u - v|.$$

Hence

$$|u - v| \geq \delta := \frac{2}{C} \tag{3}$$

for all distinct $u, v \in U_k$. Since $L_k \to \infty$, this also forces $T_k \to \infty$.

Regard each U_k as an element of $\mathcal{X}_\delta$ and define the averaged translate measure

$$\nu_k := \frac{1}{T_k} \int_0^{T_k} \delta_{T_t U_k} \, dt = \frac{1}{T_k} \int_0^{T_k} \delta_{U_k - t} \, dt$$

on $\mathcal{X}_\delta$. By compactness, pass to a weakly convergent subsequence, still denoted ν_k, with limit ν.

The limit ν is translation invariant. Indeed, for fixed $s \in \mathbb{R}$, the push-forward $(T_s)_{\#}\nu_k$ is obtained by averaging over the interval $[s, T_k + s]$ instead of $[0, T_k]$. Therefore

$$\|(T_s)_{\#}\nu_k - \nu_k\|_{\mathrm{TV}} \le \frac{2|s|}{T_k} \to 0,$$

and passing to the weak limit gives $(T_s)_{\#}\nu = \nu$.

We next compute the intensity. Let $\varphi \ge 0$ be continuous and compactly supported, with $\operatorname{supp}\varphi \subset [-R, R]$. Then

$$\int_{\mathcal{X}_\delta} N_\varphi(\Lambda)\, d\nu_k(\Lambda) = \frac{1}{T_k} \sum_{u \in U_k} \int_0^{T_k} \varphi(u - t)\, dt.$$

If $u \in [R, T_k - R]$, the inner integral equals $\int_{\mathbb{R}} \varphi(x)\, dx$. By the separation (3), the number of points of U_k lying in the two boundary intervals $[0, R]$ and $[T_k - R, T_k]$ is $O_{C,R}(1)$. Hence

$$\liminf_{k\to\infty} \int_{\mathcal{X}_\delta} N_\varphi(\Lambda)\, d\nu_k(\Lambda) \ge \liminf_{k\to\infty} \frac{L_k}{T_k} \int_{\mathbb{R}} \varphi(x)\, dx \ge \frac{1}{\pi} \int_{\mathbb{R}} \varphi(x)\, dx.$$

Since N_φ is continuous on $\mathcal{X}_\delta$, weak convergence gives

$$\int_{\mathcal{X}_\delta} N_\varphi(\Lambda)\, d\nu(\Lambda) \ge \frac{1}{\pi} \int_{\mathbb{R}} \varphi(x)\, dx. \tag{4}$$

The left side is the first moment measure of the stationary process ν. Stationarity implies that this first moment measure is translation invariant, hence it is $I\, dx$ for some $I \ge 0$. From (4),

$$I \ge \frac{1}{\pi}. \tag{5}$$

We now show that every configuration in $\operatorname{supp}\nu$ inherits the finite interpolation property. Let $\Lambda \in \operatorname{supp}\nu$, and let

$$E = \{\lambda_1, \dots, \lambda_N\} \subset \Lambda$$

be finite. Choose labels $b_1, \dots, b_N \in [-1, 1]$. Since $\Lambda \in \operatorname{supp}\nu$, every vague neighborhood of Λ has positive ν-measure. Because $\nu_k \Rightarrow \nu$, one can choose $k_j \to \infty$ and $t_j \in [0, T_{k_j}]$ such that

$$U_{k_j} - t_j \longrightarrow \Lambda$$

vaguely on larger and larger compact intervals. Taking the neighborhoods small enough and using the δ-separation, we may choose unique points

$$w_{j,r} = u_{j,r} - t_j \in U_{k_j} - t_j, \qquad r = 1, \dots, N,$$

with $w_{j,r} \to \lambda_r$.

Prescribe the value b_r at the original point $u_{j,r} \in U_{k_j}$, $r = 1, \dots, N$, and extend the data arbitrarily to the remaining points of U_{k_j}. The finite interpolation property gives $h_j \in \mathcal{B}_1$ with

$$\|h_j\|_{\infty,\mathbb{R}} \le C, \qquad h_j(u_{j,r}) = b_r, \quad r = 1, \dots, N.$$

Set

$$f_j(z) := h_j(z + t_j).$$

Then $f_j \in \mathcal{B}_1$, $\|f_j\|_{\infty,\mathbb{R}} \le C$, and $f_j(w_{j,r}) = b_r$ for $r = 1, \dots, N$. By the strip estimate (1),

$$|f_j(x+iy)| \le Ce^{|y|}, \qquad x, y \in \mathbb{R},$$

so the family is locally bounded in $\mathbb{C}$. Montel's theorem gives a locally uniformly convergent subsequence. Its limit $f \in \mathcal{B}_1$ satisfies $\|f\|_{\infty,\mathbb{R}} \le C$ and

$$f(\lambda_r) = b_r, \qquad r = 1, \dots, N.$$

Thus every finite subset of every $\Lambda \in \operatorname{supp} \nu$ is C-interpolating for real $[-1, 1]$ data.

Now enumerate a fixed $\Lambda \in \operatorname{supp} \nu$ as $(\lambda_j)_{j\ge1}$; if Λ is finite the following conclusion is interpreted only for that finite set, and later we choose an infinite configuration. Given real data (b_j) with $\sup_j |b_j| \le 1$, apply the preceding finite interpolation statement to the first N points and take a diagonal Montel limit as $N \to \infty$. This gives $f \in \mathcal{B}_1$ with

$$\|f\|_{\infty,\mathbb{R}} \le C, \qquad f(\lambda_j) = b_j \quad (j \ge 1).$$

Consequently every infinite $\Lambda \in \operatorname{supp} \nu$ interpolates all real ℓ^∞ data of norm at most one, with interpolation constant at most C. By decomposing complex data into real and imaginary parts, every such Λ interpolates complex ℓ^∞ data with interpolation constant at most $2C$.

Finally decompose the stationary probability measure ν into ergodic components. The intensity of ν is the average of the intensities of its ergodic components, so by (5) at least one component has intensity $I' \ge 1/\pi$. This component is supported on $\operatorname{supp} \nu$ up to a null set. For almost every configuration Λ in that component, the continuous-parameter Birkhoff theorem applied to the bounded observable

$$X(\Lambda) := \#(\Lambda \cap [0, 1))$$

gives

$$\lim_{R\to\infty} \frac{1}{R} \int_0^R X(T_t\Lambda)\, dt = I'.$$

The integral counts points of Λ in $[0, R]$ with weight one except for points in a boundary interval of length at most one at each end. Since all configurations are δ-separated, the boundary error is $O_\delta(1)$. Therefore

$$\lim_{R\to\infty} \frac{\#(\Lambda \cap [0, R])}{R} = I'.$$

In particular Λ is infinite and

$$D^+(\Lambda) \ge \limsup_{R\to\infty} \frac{\#(\Lambda \cap [0, R])}{R} = I' \ge \frac{1}{\pi}.$$

Since this typical Λ lies in $\operatorname{supp} \nu$, it is an interpolation sequence for $\mathcal{B}_1$. This proves the lemma. □

Proof of Proposition 3.1. If $0 < C < 1$, take $L = 1$ and any $\eta > 0$, and prescribe the single label $a_1 = 1$. No function of real-line sup norm at most C can attain this value. Hence assume $C \ge 1$.

Suppose the proposition fails. Then for every $\eta > 0$ and every $L \ge 1$ there is an L-point multiset

$$U \subset \mathbb{R}, \qquad \operatorname{diam} U \le \pi(1 + \eta)L,$$

such that every assignment of labels in $[-1, 1]$ is interpolable on U by some complex-valued $f \in \mathcal{B}_1$ of norm at most C.

Such a multiset cannot contain repeated points, because assigning labels 1 and -1 to two copies of the same point would be impossible. Thus it is a set. Choose $\eta_k \downarrow 0$ and $L_k \to \infty$, and pick corresponding sets U_k. Translate each set so that its convex hull is $[0, T_k]$; then $\#U_k = L_k$ and $T_k \le \pi(1 + \eta_k)L_k$. Lemma 3.2 gives a separated interpolation sequence Λ for $\mathcal{B}_1$ with $D^+(\Lambda) \ge 1/\pi$. This contradicts Theorem 2.2. Therefore the proposition holds. □

4 Angular blocking of algebraic nodes

We now turn the finite Bernstein obstruction into labels for algebraic nodes.

Fix $C > 0$, and let $\eta > 0$ and $L \in \mathbb{N}$ be given by Proposition 3.1. Choose

$$0 < \varepsilon < \min\left\{\frac{1}{2}, \frac{\eta}{2\bigl(1 + (1+\eta)L\bigr)}\right\}. \tag{6}$$

This choice implies

$$\frac{\eta - \varepsilon}{(1+\eta)L} > \varepsilon. \tag{7}$$

Let n be large. Define

$$D_n := \lceil (1+\varepsilon)n \rceil.$$

Given nodes $x_1, \dots, x_n \in [-1, 1]$, choose their angular representatives

$$\theta_i := \arccos x_i \in [0, \pi].$$

Reorder the indices, only for notational convenience, so that

$$0 \le \theta_1 \le \theta_2 \le \cdots \le \theta_n \le \pi.$$

The final labels are then assigned back to the original indices.

Partition the ordered list into consecutive full blocks of size L:

$$B_b := \{(b-1)L + 1, \dots, bL\}, \qquad b = 1, \dots, N,$$

where

$$N := \left\lfloor \frac{n}{L} \right\rfloor.$$

There are fewer than L leftover indices; ignore them for the blocking count. The angular span of the block B_b is

$$h_b := \theta_{bL} - \theta_{(b-1)L+1} \ge 0.$$

Call B_b good if

$$D_n h_b \le \pi(1+\eta)L. \tag{8}$$

Lemma 4.1 (Many good blocks)**.** *For all sufficiently large n, the number G of good full blocks satisfies*

$$G > \varepsilon n.$$

Proof. The intervals

$$[\theta_{(b-1)L+1}, \theta_{bL}], \qquad b = 1, \dots, N,$$

are consecutive and have disjoint interiors inside $[0, \pi]$. Therefore

$$\sum_{b=1}^{N} h_b \le \pi.$$

If B is the number of bad blocks, then every bad block satisfies

$$h_b > \frac{\pi(1+\eta)L}{D_n},$$

and hence

$$B < \frac{D_n}{(1+\eta)L}.$$

Using $D_n \le (1+\varepsilon)n + 1$,

$$\begin{aligned} G &= N - B \\ &\ge \left\lfloor \frac{n}{L} \right\rfloor - \frac{D_n}{(1+\eta)L} \\ &\ge \frac{n}{L} - 1 - \frac{(1+\varepsilon)n+1}{(1+\eta)L} \\ &= n\frac{\eta-\varepsilon}{(1+\eta)L} - 1 - \frac{1}{(1+\eta)L}. \end{aligned}$$

By (7), the coefficient of n on the last line is strictly larger than ε. Thus $G > \varepsilon n$ once n is sufficiently large. □

We now assign the labels. On every good block B_b, define scaled local nodes

$$u_{b,j} := D_n\big(\theta_{(b-1)L+j} - \theta_{(b-1)L+1}\big), \qquad j = 1, \dots, L.$$

By goodness,

$$\operatorname{diam}\{u_{b,1}, \dots, u_{b,L}\} \le \pi(1+\eta)L.$$

Proposition 3.1 supplies labels

$$a_{b,1}, \dots, a_{b,L} \in [-1, 1]$$

which cannot be interpolated at the nodes $u_{b,j}$ by any $f \in \mathcal{B}_1$ of real-line sup norm at most C. Define

$$y_{(b-1)L+j} := a_{b,j}$$

on good blocks. On bad blocks and leftover indices, set $y_i = 0$.

5 Proof of the main theorem

We finish the proof of Theorem 1.1. Assume, toward contradiction, that P is a polynomial satisfying

$$\deg P = m < (1+\varepsilon)n, \qquad \|P\|_{L^\infty[-1,1]} \le C,$$

and that P fits at least $(1-\varepsilon)n$ of the labels constructed above.

Because $D_n = \lceil (1+\varepsilon)n \rceil$, the strict degree inequality implies

$$m < D_n. \tag{9}$$

Let

$$Q(\theta) := P(\cos\theta).$$

The Chebyshev expansion of P shows that Q is an even trigonometric polynomial of degree at most m. In particular Q is an entire function of exponential type at most m, and

$$\|Q\|_{\infty,\mathbb{R}} = \sup_{\theta\in\mathbb{R}} |P(\cos\theta)| = \|P\|_{L^\infty[-1,1]} \le C.$$

Fix a good block B_b. Define the rescaled function

$$F_b(u) := Q\left(\theta_{(b-1)L+1} + \frac{u}{D_n}\right).$$

Then F_b is entire of exponential type at most $m/D_n < 1$ by (9), and

$$\|F_b\|_{\infty,\mathbb{R}} \le C.$$

Thus $F_b \in \mathcal{B}_1$ with norm at most C.

If P fit every label in this good block, then for each $j = 1, \dots, L$,

$$\begin{aligned} F_b(u_{b,j}) &= Q\left(\theta_{(b-1)L+1} + \frac{D_n(\theta_{(b-1)L+j} - \theta_{(b-1)L+1})}{D_n}\right) \\ &= Q(\theta_{(b-1)L+j}) \\ &= P(x_{(b-1)L+j}) \\ &= a_{b,j}. \end{aligned}$$

This contradicts the choice of the labels $a_{b,j}$ from Proposition 3.1. Therefore P must miss at least one label in every good block.

By Lemma 4.1, the number of good blocks is greater than εn for all sufficiently large n. Hence P misses more than εn labels. This contradicts the assumption that P fits at least $(1-\varepsilon)n$ labels. The contradiction proves the theorem.

6 Checks, variants, and constants

6.1 Why the strict inequality in Beurling's theorem is enough

The finite obstruction does not require a quantitative estimate of the interpolation constants near critical density. If uniformly C-interpolating finite blocks existed at densities tending to $1/\pi$ from below, the stationary compactness argument would produce an infinite sequence of density at least $1/\pi$ with bounded interpolation constant. The strict inequality in Theorem 2.2 rules this out.

6.2 Complex polynomials

The argument proves a slightly stronger statement than the usual real-polynomial formulation: even complex polynomials cannot fit the labels while remaining bounded by C. The finite forbidden labels are real, but Proposition 3.1 excludes complex-valued Bernstein functions as well.

6.3 Dependence of ε on C

The proof is qualitative. The constants $L(C)$ and $\eta(C)$ are obtained by compactness from Beurling's strict density theorem, so no explicit numerical value of $\varepsilon(C)$ is extracted. Once such L and η are known, any

$$0 < \varepsilon < \frac{\eta}{2(1+(1+\eta)L)}$$

works after increasing $n_0(C)$ to absorb the $O(1)$ losses in the block count.

6.4 Relation to Lagrange-interpolation probes

A direct Lagrange-interpolation computation at the fitted nodes can be misleading for this problem, because the polynomial is allowed degree $< (1+\varepsilon)n$ while it need only fit about $(1-\varepsilon)n$ points. Locally, after fitting a block, the additional degrees of freedom correspond to a Bernstein-space correction. The proof above avoids this issue by using the true near-critical interpolation obstruction in the local Bernstein model.

7 Conclusion

The robust obstruction is a consequence of a simple density principle: near the critical Bernstein interpolation density, uniformly bounded interpolation of all local data is impossible. The angular substitution $x = \cos\theta$ transfers this local fact from entire functions of exponential type to algebraic polynomials on $[-1, 1]$. A block pigeonhole argument then turns a local obstruction into a global robust obstruction for arbitrary node sets.

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