A Threshold Consistency Theorem for Erdős Problem 598

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Abstract

Let

$$\kappa = (2^{\aleph_0})^+.$$

Erdős asked whether for every infinite cardinal m there exists a coloring

$$c\colon [m]^\omega \to \kappa$$

such that every $X \in [m]^\kappa$ contains countable subsets of all κ colors. We combine four ingredients.

First, we record the repaired forum proof of the base case $m = \kappa$. Second, we prove a new *thick/thin disjoint-sum theorem*: if $\mathsf{Col}_\omega(\theta, \kappa)$ and $\mathsf{Col}_\omega(A_i, \kappa)$ hold for all $i < \theta$, then

$$\mathsf{Col}_\omega\Big(\bigsqcup_{i<\theta} A_i, \kappa\Big)$$

holds. Third, we include a countable-product closure theorem, yielding $\mathsf{Col}_\omega(m, \kappa)$ for every $m \leq \kappa^\omega$. Fourth, we combine a bounded-set obstruction with results of Garti and Hayut on Magidor cardinals to obtain a relative-consistency counterexample.

The structural consequence is clean. In any model in which a counterexample exists, there is a least bad cardinal λ_*; our new sum theorem shows that

$$\lambda_* > \kappa^\omega \qquad \text{and} \qquad \mathrm{cf}(\lambda_*) > \kappa.$$

Moreover $\mathsf{Col}_\omega(m, \kappa)$ holds for every $m < \lambda_*$ and fails for every $m \geq \lambda_*$. Assuming suitable large-cardinal hypotheses, there is a forcing extension with such a threshold cardinal λ_*. Thus Erdős Problem 598 admits a complete yes/no classification in that model, even though the absolute ZFC classification remains open.

1 Introduction

In [3], Erdős asked whether the following holds.

Erdős Problem 598. Let m be an infinite cardinal and let

$$\kappa = (2^{\aleph_0})^+.$$

Does there exist a coloring

$$c\colon [m]^\omega \to \kappa$$

such that for every $X \in [m]^\kappa$ one has

$$c''[X]^\omega = \kappa?$$

The problem is recorded on the Erdős problem site in [1] and, as of April 21, 2026, is still listed there as open. A forum discussion [2] contains a direct stationary-set argument for the special case $m = \kappa$. The only issue in that post is that the coloring is explicitly defined only on those countable sets whose supremum has cofinality ω; assigning any default color on the remaining countable sets repairs the proof.

To organize the discussion, we isolate the coloring property.

Definition 1.1. Let A be a set and let κ be an infinite cardinal. We write $\mathsf{Col}_\omega(A, \kappa)$ if there exists a coloring

$$c\colon [A]^\omega \to \kappa$$

such that

$$c''[X]^\omega = \kappa$$

for every $X \in [A]^\kappa$. If $|A| = \lambda$, then we also write $\mathsf{Col}_\omega(\lambda, \kappa)$. Equivalently,

$$\mathsf{Col}_\omega(\lambda, \kappa) \iff \lambda \not\to [\kappa]^\omega_\kappa.$$

We also write

$$[\lambda]^{\omega\text{-bd}} = \{a \in [\lambda]^\omega : \sup(a) < \lambda\}$$

for the family of bounded countably infinite subsets of an ordinal λ.

The paper has two themes. The first is positive: we prove several closure theorems for $\mathsf{Col}_\omega(-, \kappa)$, including a new thick/thin sum theorem and the countable-product theorem from the earlier note. The second is negative: using Magidor cardinals, we show that under suitable large-cardinal assumptions there is a model in which the property eventually fails. Combining these gives a threshold picture.

2 Basic positive constructions

We start with the forum argument in the regular base case.

Proposition 2.1. *Let κ be a regular uncountable cardinal. Then $\mathsf{Col}_\omega(\kappa, \kappa)$ holds.*

Proof. Let

$$E^\kappa_\omega = \{\delta < \kappa : \mathrm{cf}(\delta) = \omega\}.$$

This set is stationary in κ. By Solovay's partition theorem [7], fix a partition

$$E^\kappa_\omega = \bigsqcup_{\xi<\kappa} S_\xi$$

with each S_ξ stationary. Define $c\colon [\kappa]^\omega \to \kappa$ by

$$c(a) = \begin{cases} \xi, & \text{if } \sup(a) \in S_\xi, \\ 0, & \text{if } \sup(a) \notin E^\kappa_\omega. \end{cases}$$

This is the repair missing from the forum post: the coloring must be total on all of $[\kappa]^\omega$.

Now let $X \in [\kappa]^\kappa$ and fix $\xi < \kappa$. Since κ is regular, X is unbounded in κ. Hence the set of limit points of X is club in κ. Choose

$$\delta \in S_\xi$$

which is also a limit point of X. Because $\mathrm{cf}(\delta) = \omega$, there exists a countably infinite $a \subseteq X \cap \delta$ cofinal in δ. Then $\sup(a) = \delta \in S_\xi$, so $c(a) = \xi$. Since $\xi < \kappa$ was arbitrary, $c''[X]^\omega = \kappa$. □

We record two elementary monotonicity properties.

Lemma 2.2 (Downward monotonicity)**.** *Let A, B be sets and let κ be an infinite cardinal. If A injects into B and $\mathsf{Col}_\omega(B, \kappa)$ holds, then $\mathsf{Col}_\omega(A, \kappa)$ holds. In particular, if $\lambda \leq \mu$ and $\mathsf{Col}_\omega(\mu, \kappa)$ holds, then $\mathsf{Col}_\omega(\lambda, \kappa)$ holds.*

Proof. Fix an injection $e \colon A \to B$ and a witness $c \colon [B]^\omega \to \kappa$ for $\mathsf{Col}_\omega(B, \kappa)$. Define

$$d(a) = c(e''a) \qquad (a \in [A]^\omega).$$

If $X \in [A]^\kappa$, then $e''X \in [B]^\kappa$, so

$$d''[X]^\omega = c''[e''X]^\omega = \kappa.$$

Thus d witnesses $\mathsf{Col}_\omega(A, \kappa)$. □

Corollary 2.3 (Upward monotonicity of failure)**.** *Let $\lambda \leq \mu$ be infinite cardinals. If $\mathsf{Col}_\omega(\lambda, \kappa)$ fails, then $\mathsf{Col}_\omega(\mu, \kappa)$ fails.*

Proof. This is the contrapositive of Lemma 2.2. □

The next theorem is the main new positive tool. It mixes two kinds of countable subsets: the *thick* ones, which have infinitely many points in one piece, and the *thin* ones, which meet every piece only finitely often.

Theorem 2.4 (Thick/thin disjoint sums)**.** *Let κ be a regular uncountable cardinal and let θ be an ordinal. Assume $\mathsf{Col}_\omega(\theta, \kappa)$, and let $\langle A_i : i < \theta \rangle$ be pairwise disjoint sets such that $\mathsf{Col}_\omega(A_i, \kappa)$ holds for every $i < \theta$. Then*

$$\mathsf{Col}_\omega\Big(\bigsqcup_{i<\theta} A_i, \kappa\Big)$$

holds.

Proof. Fix a witness

$$d \colon [\theta]^\omega \to \kappa$$

for $\mathsf{Col}_\omega(\theta, \kappa)$, and for each $i < \theta$ fix a witness

$$c_i \colon [A_i]^\omega \to \kappa$$

for $\mathsf{Col}_\omega(A_i, \kappa)$. Let

$$A = \bigsqcup_{i<\theta} A_i.$$

We define $c \colon [A]^\omega \to \kappa$ by a thick/thin dichotomy.

Given $a \in [A]^\omega$, if there exists $i < \theta$ with $|a \cap A_i| = \omega$, let $i(a)$ be the least such i and set

$$c(a) = c_{i(a)}(a \cap A_{i(a)}).$$

If no such i exists, then every intersection $a \cap A_i$ is finite, so the support

$$\operatorname{supp}(a) = \{i < \theta : a \cap A_i \neq \varnothing\}$$

is a countably infinite subset of θ. In this case define

$$c(a) = d(\operatorname{supp}(a)).$$

We claim that c witnesses $\mathsf{Col}_\omega(A, \kappa)$. Fix $X \in [A]^\kappa$ and $\xi < \kappa$. There are two cases.

Case 1. For some $i < \theta$ one has $|X \cap A_i| = \kappa$. Choose $a \in [X \cap A_i]^\omega$ with $c_i(a) = \xi$. Then $i(a) = i$, so $c(a) = \xi$.

Case 2. For every $i < \theta$ one has $|X \cap A_i| < \kappa$. Let

$$I(X) = \{i < \theta : X \cap A_i \neq \varnothing\}.$$

We claim that $|I(X)| \geq \kappa$. Indeed, if $|I(X)| < \kappa$, then by the regularity of κ,

$$|X| \leq \sum_{i \in I(X)} |X \cap A_i| < \kappa,$$

contradicting $|X| = \kappa$. Choose some $Y \in [I(X)]^\kappa$. Since d witnesses $\mathsf{Col}_\omega(\theta, \kappa)$, pick $J \in [Y]^\omega$ such that

$$d(J) = \xi.$$

For each $j \in J$, choose one point $x_j \in X \cap A_j$, and set

$$a = \{x_j : j \in J\}.$$

Then $a \in [X]^\omega$, every intersection $a \cap A_i$ is finite, and $\mathrm{supp}(a) = J$. Hence

$$c(a) = d(J) = \xi.$$

Since $\xi < \kappa$ was arbitrary, we conclude that $c''[X]^\omega = \kappa$. □

Corollary 2.5 (Closure under $< \kappa$ disjoint sums)**.** *Let κ be a regular uncountable cardinal and let $\theta < \kappa$. If $\langle A_i : i < \theta \rangle$ is a pairwise disjoint family and $\mathsf{Col}_\omega(A_i, \kappa)$ holds for every $i < \theta$, then*

$$\mathsf{Col}_\omega\Big(\bigsqcup_{i<\theta} A_i, \kappa\Big)$$

holds.

Proof. Since $\theta < \kappa$, the statement $\mathsf{Col}_\omega(\theta, \kappa)$ is vacuous. Apply Theorem 2.4. □

We also need the countable-product theorem from the earlier note.

Theorem 2.6 (Closure under countable products)**.** *Let κ be a regular uncountable cardinal such that*

$$\mu^\omega < \kappa \qquad \text{for every } \mu < \kappa.$$

Let $\langle A_n : n < \omega \rangle$ be a sequence of sets. If $\mathsf{Col}_\omega(A_n, \kappa)$ holds for every $n < \omega$, then

$$\mathsf{Col}_\omega\Big(\prod_{n<\omega} A_n, \kappa\Big)$$

holds.

Proof. For each $n < \omega$, fix a witness

$$c_n \colon [A_n]^\omega \to \kappa.$$

Let

$$A = \prod_{n<\omega} A_n,$$

and for each $n < \omega$ let $\pi_n \colon A \to A_n$ be the nth coordinate map. Define $c \colon [A]^\omega \to \kappa$ as follows. Given $a \in [A]^\omega$, let $n(a)$ be the least $n < \omega$ such that $|\pi_n[a]| = \omega$, if such an n exists. If $n(a)$ exists, set

$$c(a) = c_{n(a)}(\pi_{n(a)}[a]).$$

If no such $n(a)$ exists, set $c(a) = 0$.

We claim that c witnesses $\mathsf{Col}_\omega(A, \kappa)$. Fix $X \in [A]^\kappa$ and $\xi < \kappa$. First, there exists some $n < \omega$ such that $|\pi_n[X]| = \kappa$. Indeed, if $|\pi_n[X]| < \kappa$ for every n, then by regularity of κ the cardinal

$$\mu := \sup_{n<\omega} |\pi_n[X]|$$

satisfies $\mu < \kappa$, and therefore

$$|X| \le \prod_{n<\omega} |\pi_n[X]| \le \mu^\omega < \kappa,$$

contradicting $|X| = \kappa$. Let n^* be the least n with $|\pi_n[X]| = \kappa$.

Consider the equivalence relation E on X given by

$$x \, E \, y \iff x \restriction n^* = y \restriction n^*.$$

The number of E-classes is at most

$$\prod_{i<n^*} |\pi_i[X]| < \kappa,$$

because each factor is $< \kappa$ by minimality of n^* and there are only finitely many factors. If every E-class Y satisfied $|\pi_{n^*}[Y]| < \kappa$, then

$$|\pi_{n^*}[X]| < \kappa$$

would follow by taking a union of $< \kappa$ many sets of size $< \kappa$, contradicting the choice of n^*. Hence there is an E-class $Y \subseteq X$ such that

$$|\pi_{n^*}[Y]| = \kappa.$$

In particular, all coordinates $< n^*$ are constant on Y.

Now apply $\mathsf{Col}_\omega(A_{n^*}, \kappa)$ to $\pi_{n^*}[Y] \in [A_{n^*}]^\kappa$. Choose $b \in [\pi_{n^*}[Y]]^\omega$ such that

$$c_{n^*}(b) = \xi.$$

For each $\beta \in b$, choose one $x_\beta \in Y$ with $\pi_{n^*}(x_\beta) = \beta$, and set

$$a = \{x_\beta : \beta \in b\}.$$

Then $a \in [X]^\omega$, the coordinates $< n^*$ are constant on a, and $\pi_{n^*}[a] = b$. Hence $n(a) = n^*$, so

$$c(a) = c_{n^*}(\pi_{n^*}[a]) = c_{n^*}(b) = \xi.$$

Since $\xi < \kappa$ was arbitrary, $c''[X]^\omega = \kappa$. □

Corollary 2.7. *Let κ be a regular uncountable cardinal such that $\mu^\omega < \kappa$ for every $\mu < \kappa$. If $\mathsf{Col}_\omega(A, \kappa)$ holds, then $\mathsf{Col}_\omega(A^\omega, \kappa)$ holds. In particular, if $\mathsf{Col}_\omega(\lambda, \kappa)$ holds, then $\mathsf{Col}_\omega(\lambda^\omega, \kappa)$ holds.*

Proof. Apply Theorem 2.6 to the constant sequence $A_n = A$. □

We now specialize to Erdős' cardinal.

Corollary 2.8. *Let $\kappa = (2^{\aleph_0})^+$. Then $\mathsf{Col}_\omega(m, \kappa)$ holds for every cardinal $m \le \kappa^\omega$.*

Proof. If $\mu < \kappa$, then by definition of κ we have $\mu \le 2^{\aleph_0}$, and therefore

$$\mu^\omega \le (2^{\aleph_0})^\omega = 2^{\aleph_0} < \kappa.$$

So the hypothesis of Corollary 2.7 is satisfied. By Proposition 2.1, $\mathsf{Col}_\omega(\kappa, \kappa)$ holds. Hence Corollary 2.7 gives $\mathsf{Col}_\omega(\kappa^\omega, \kappa)$. Finally, Lemma 2.2 yields $\mathsf{Col}_\omega(m, \kappa)$ for every $m \le \kappa^\omega$. □

3 Threshold cardinals if failure exists

This section records the main structural consequence of the new sum theorem. Suppose that, for the fixed color number κ, the Erdős property eventually fails. Then there is a least bad cardinal, and it is forced into a narrow range.

Theorem 3.1 (Threshold structure)**.** *Fix $\kappa = (2^{\aleph_0})^+$. Assume there exists an infinite cardinal λ such that $\mathsf{Col}_\omega(\lambda, \kappa)$ fails. Let*

$$\lambda_* = \min\{\lambda \ge \kappa : \neg\mathsf{Col}_\omega(\lambda, \kappa)\}.$$

Then the following hold.

(a) $\mathsf{Col}_\omega(m, \kappa)$ *holds for every* $m < \lambda_*$.

(b) $\mathsf{Col}_\omega(m, \kappa)$ *fails for every* $m \ge \lambda_*$.

(c) $\lambda_* > \kappa^\omega$.

(d) $\mathrm{cf}(\lambda_*) > \kappa$.

Proof. Part (a) is the definition of λ_*. Part (b) follows from Corollary 2.3. Part (c) follows from Corollary 2.8.

It remains to prove part (d). Suppose first that $\mathrm{cf}(\lambda_*) < \kappa$. Choose an increasing continuous cofinal sequence

$$\langle \lambda_i : i < \mathrm{cf}(\lambda_*) \rangle$$

in λ_*, and set

$$A_i = [\lambda_i, \lambda_{i+1})$$

for $i < \mathrm{cf}(\lambda_*)$. Then the sets A_i are pairwise disjoint, their union is λ_*, and each $|A_i| < \lambda_*$. By minimality of λ_*, each A_i satisfies $\mathsf{Col}_\omega(A_i, \kappa)$. Since $\mathrm{cf}(\lambda_*) < \kappa$, also $\mathsf{Col}_\omega(\mathrm{cf}(\lambda_*), \kappa)$ holds vacuously. Applying Theorem 2.4, we get $\mathsf{Col}_\omega(\lambda_*, \kappa)$, a contradiction.

Now suppose that $\mathrm{cf}(\lambda_*) = \kappa$. Again choose an increasing continuous cofinal sequence

$$\langle \lambda_i : i < \kappa \rangle$$

in λ_* and set $A_i = [\lambda_i, \lambda_{i+1})$. As above, each A_i satisfies $\mathsf{Col}_\omega(A_i, \kappa)$ by minimality. Proposition 2.1 gives $\mathsf{Col}_\omega(\kappa, \kappa)$. Therefore Theorem 2.4 implies $\mathsf{Col}_\omega(\lambda_*, \kappa)$, again a contradiction.

Hence neither $\mathrm{cf}(\lambda_*) < \kappa$ nor $\mathrm{cf}(\lambda_*) = \kappa$ is possible, so $\mathrm{cf}(\lambda_*) > \kappa$. □

Remark 3.2. Theorem 3.1 shows that any first ZFC counterexample, if one exists, must lie strictly above κ^ω and must have cofinality strictly larger than κ. This pinpoints the unresolved range. The thick/thin sum theorem settles all disjoint-sum constructions of cofinality at most κ, but it is not coherent on initial segments, so it does not by itself iterate through higher-cofinality stages.

4 A bounded-set obstruction and a threshold model

We now turn to the negative side. The next theorem converts any omitting result on bounded countable subsets into a failure of $\mathsf{Col}_\omega(\lambda, \kappa)$.

Theorem 4.1 (Bounded obstruction)**.** *Let λ be a singular cardinal and let κ be a regular uncountable cardinal such that*

$$\mathrm{cf}(\lambda) < \kappa \leq \lambda.$$

Assume

$$\lambda \to [\lambda]^{\omega\text{-}bd}_\kappa.$$

Then $\mathsf{Col}_\omega(\lambda, \kappa)$ fails. More precisely, for every coloring $c\colon [\lambda]^\omega \to \kappa$ there exist a color $\xi < \kappa$ and a bounded set $X \in [\lambda]^\kappa$ such that

$$\xi \notin c''[X]^\omega.$$

Equivalently,

$$\lambda \to [\kappa]^\omega_\kappa.$$

Proof. Fix any coloring

$$c\colon [\lambda]^\omega \to \kappa.$$

Restrict it to bounded countable subsets:

$$c_{\mathrm{bd}} = c \restriction [\lambda]^{\omega\text{-bd}}.$$

By the hypothesis $\lambda \to [\lambda]^{\omega\text{-bd}}_\kappa$, there exist $A \in [\lambda]^\lambda$ and $\xi < \kappa$ such that

$$\xi \notin c''_{\mathrm{bd}}[A]^{\omega\text{-bd}}.$$

Choose an increasing cofinal sequence

$$\langle \lambda_i : i < \mathrm{cf}(\lambda) \rangle$$

in λ. We claim that for some $i < \mathrm{cf}(\lambda)$,

$$|A \cap \lambda_i| \geq \kappa.$$

Indeed, if $|A \cap \lambda_i| < \kappa$ for every $i < \mathrm{cf}(\lambda)$, then by regularity of κ and the inequality $\mathrm{cf}(\lambda) < \kappa$,

$$|A| \leq \sum_{i<\mathrm{cf}(\lambda)} |A \cap \lambda_i| < \kappa,$$

contradicting $|A| = \lambda \geq \kappa$.

Fix $i_* < \mathrm{cf}(\lambda)$ with $|A \cap \lambda_{i_*}| \geq \kappa$, and choose

$$X \in [A \cap \lambda_{i_*}]^\kappa.$$

Then X is bounded in λ. Hence every $a \in [X]^\omega$ belongs to $[A]^{\omega\text{-bd}}$, so

$$c(a) = c_{\mathrm{bd}}(a) \neq \xi.$$

Therefore $\xi \notin c''[X]^\omega$, as required. □

Recall that an infinite cardinal λ is *Magidor* if

$$\lambda \to [\lambda]^{\omega\text{-bd}}_{\lambda}.$$

If λ is Magidor, the least ordinal $\alpha < \lambda$ such that

$$\lambda \to [\lambda]^{\omega\text{-bd}}_{\alpha}$$

is denoted by $\alpha_M(\lambda)$. The next corollary is immediate.

Corollary 4.2. *Let λ be a Magidor cardinal and let κ be a regular uncountable cardinal such that*

$$\alpha_M(\lambda) \le \kappa < \lambda.$$

Then $\mathsf{Col}_\omega(\lambda, \kappa)$ fails.

Proof. Since λ is Magidor, one has $\mathrm{cf}(\lambda) = \omega$; see [5]. Also, by definition of $\alpha_M(\lambda)$, one has

$$\lambda \to [\lambda]^{\omega\text{-bd}}_{\alpha_M(\lambda)}.$$

If $\kappa \ge \alpha_M(\lambda)$, then also

$$\lambda \to [\lambda]^{\omega\text{-bd}}_{\kappa}$$

holds. Indeed, given any coloring into κ colors, compose it with a surjection from κ onto $\alpha_M(\lambda)$. Now apply Theorem 4.1. ☐

We now combine this with Garti–Hayut's forcing theorem.

Theorem 4.3 (Threshold model from Magidority)**.** *Assume suitable large-cardinal hypotheses. Then there is a forcing extension in which the following all hold.*

(a) $\kappa = (2^{\aleph_0})^+ = \aleph_2$.

(b) There exists a cardinal $\lambda_* > \kappa^\omega$ with $\mathrm{cf}(\lambda_*) > \kappa$ such that

$$\mathsf{Col}_\omega(m, \kappa) \quad \textit{holds iff} \quad m < \lambda_*.$$

Proof. By [6, Claim 1.10(a)], assuming suitable large-cardinal hypotheses, there is a forcing extension with some Magidor cardinal λ satisfying

$$\alpha_M(\lambda) = \aleph_2.$$

By [5, Claim 1.3(c) and Corollary 1.6], every Magidor cardinal satisfies

$$2^{\aleph_0} < \alpha_M(\lambda).$$

Hence in this extension,

$$2^{\aleph_0} < \aleph_2,$$

so necessarily

$$2^{\aleph_0} = \aleph_1 \qquad \text{and therefore} \qquad \kappa = (2^{\aleph_0})^+ = \aleph_2.$$

Since $\alpha_M(\lambda) = \kappa$, Corollary 4.2 gives

$$\neg\mathsf{Col}_\omega(\lambda, \kappa).$$

Thus some bad cardinal exists. Let λ_* be the least bad cardinal. By Theorem 3.1,

$$\lambda_* > \kappa^\omega \qquad \text{and} \qquad \mathrm{cf}(\lambda_*) > \kappa,$$

and moreover

$$\mathsf{Col}_\omega(m, \kappa) \quad \text{holds iff} \quad m < \lambda_*.$$

This proves the theorem. ☐

Remark 4.4. Theorem 4.3 gives a complete yes/no classification for every cardinal m in a forcing extension. It does *not* identify the threshold cardinal λ_* explicitly, and it does not settle the original ZFC question on the Erdős problem page. What it does show is that any absolute ZFC proof or disproof must confront the regime described in Remark 3.2.

References

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