

# Erdős Problem #655 and Its Natural Repairs: Exact Resolutions, Historical Sources, and Open Variants

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## Abstract

We give a consolidated account of the current formulation of Erdős Problem #655 and the nearby variants suggested on the forum and in the older literature. Let  $D(X)$  denote the number of distinct pairwise distances determined by a finite planar set  $X$ , let  $d_X(x)$  be the number of distinct distances from a point  $x \in X$ , let  $M(X) := \max_{x \in X} d_X(x)$ , and let  $\Sigma(X) := \sum_{x \in X} d_X(x)$ . For the local condition  $\mathcal{A}_2$  that every circle centered at a point of  $X$  contains at most two other points of  $X$ , we prove the exact sharp formulas

$$\min D(X) = \min M(X) = \left\lfloor \frac{n}{2} \right\rfloor, \quad \min \Sigma(X) = n \left\lfloor \frac{n}{2} \right\rfloor,$$

all attained by the regular  $n$ -gon. Hence the literal statement of Problem #655, and all its obvious “repairs” that still admit the regular  $n$ -gon (for instance adding only “no three collinear” or “convex position”), are false in the strongest possible way.

We then explain how this fits the older Erdős sources. In 1987 Erdős asked *general-position* questions at the natural  $n/3$  scale, both for the global number of distances and for the pinned quantity  $M(X)$ ; in 1988 he asked the stronger pinned  $n/2$  question under the stronger local condition  $\mathcal{A}_2$  together with a cocircularity restriction. This leads to a clean taxonomy of variants. We record which versions are completely settled, which reduce to classical open problems (the no-three-collinear problem, the general-position problem, and the convex pinned problem), and which remain the most faithful open repairs of the literal website statement.

## 1 Introduction

The current page for Erdős Problem #655 on *erdosproblems.com* asks whether an  $n$ -point set  $X \subset \mathbb{R}^2$  satisfying the local condition

no circle whose center is one of the points of  $X$  contains three other points of  $X$

must determine at least  $(1+c)\frac{n}{2}$  distinct pairwise distances for some fixed  $c > 0$  and all large  $n$  [4]. The same page already records Zach Hunter’s observation that a regular  $n$ -gon is a counterexample, and suggests that some additional general-position assumption was probably intended. The discussion thread from 19 January 2026 explicitly mentions both

- (i) general position (“no three on a line and no four on a circle”), and
- (ii) convex position,

as natural candidate repairs, before the database was marked as containing an ambiguous statement [5].

The goal of this note is to organize all of these variants at once.

First, we settle exactly the literal statement of Problem #655 and all versions that still admit the regular  $n$ -gon. In particular, adding only “no three on a line” or “convex position” does *not*

repair the problem: the same regular-polygon example remains extremal. We also settle the pinned and summed analogues under the same local hypothesis.

Second, we trace the older source material. The forum's general-position suggestion is historically well justified: Erdős already asked such questions in 1987 [10]. The convex-position suggestion is also historically natural: the global convex version goes back to Erdős' 1946 paper and was proved by Altman, while the corresponding pinned convex version remains open [9, 1, 2, 12, 6, 16]. What emerges is that there are really *three* distinct families of problems nearby:

- (1) the literal local- $\mathcal{A}_2$  questions at the  $n/2$  scale;
- (2) the no-three-collinear and general-position questions at the  $n/3$  scale;
- (3) the convex-position questions, where the global problem is solved but the pinned one is still open.

## 2 Notation and local hypotheses

Let  $X \subset \mathbb{R}^2$  be a finite set of size  $n$ . We write

$$D(X) := |\{|x - y| : x, y \in X, x \neq y\}|$$

for the number of distinct pairwise distances determined by  $X$ . For  $x \in X$  we write

$$d_X(x) := |\{|x - y| : y \in X \setminus \{x\}\}|$$

for the number of distinct distances from  $x$  to the other points of  $X$ , and we set

$$M(X) := \max_{x \in X} d_X(x), \quad \Sigma(X) := \sum_{x \in X} d_X(x).$$

Thus  $M(X)$  is the usual pinned quantity, while  $\Sigma(X)$  is the sum-version considered by Erdős in the same circle of questions.

For  $m \geq 1$ , let  $\mathcal{A}_m$  denote the class of finite planar sets  $X$  with the property that every circle centered at a point of  $X$  contains at most  $m$  other points of  $X$ . Thus the literal hypothesis of Problem #655 is exactly the condition  $X \in \mathcal{A}_2$ .

We will also use the following standard classes:

$$\begin{aligned} \mathcal{N}_3 &:= \{X \subset \mathbb{R}^2 : \text{no three points of } X \text{ are collinear}\}, \\ \mathcal{N}_4 &:= \{X \subset \mathbb{R}^2 : \text{no four points of } X \text{ are cocircular}\}, \\ \mathcal{G} &:= \mathcal{N}_3 \cap \mathcal{N}_4 \quad (\text{general position}), \\ \mathcal{C} &:= \{X \subset \mathbb{R}^2 : X \text{ is the vertex set of a convex polygon}\}. \end{aligned}$$

The following trivial pigeonhole lemma explains why the constants  $1/2$  and  $1/3$  appear throughout the older literature.

**Lemma 2.1.** *If  $X \in \mathcal{A}_m$  and  $|X| = n$ , then for every  $x \in X$  one has*

$$d_X(x) \geq \left\lceil \frac{n-1}{m} \right\rceil.$$

Consequently,

$$M(X) \geq \left\lceil \frac{n-1}{m} \right\rceil, \quad D(X) \geq \left\lceil \frac{n-1}{m} \right\rceil, \quad \Sigma(X) \geq n \left\lceil \frac{n-1}{m} \right\rceil.$$

*Proof.* Fix  $x \in X$ . The  $n - 1$  distances from  $x$  to  $X \setminus \{x\}$  come in classes of equal values. By definition of  $\mathcal{A}_m$ , each class has size at most  $m$ . Hence at least  $\lceil (n - 1)/m \rceil$  classes are needed.  $\square$

*Remark 2.2.* If  $X \in \mathcal{N}_4$ , then  $X \in \mathcal{A}_3$ : a circle centered at  $x \in X$  that contained four other points of  $X$  would contain four cocircular points of  $X$ . Thus every no-four-cocircular set satisfies

$$d_X(x), M(X), D(X) \geq \left\lceil \frac{n-1}{3} \right\rceil.$$

This is the trivial  $n/3$  lower bound that appears in the general-position and pinned variants [10, 3].

### 3 The exact local- $\mathcal{A}_2$ theorem

The next theorem settles the literal statement of Problem #655 and, simultaneously, the pinned and sum versions under the same hypothesis.

**Theorem 3.1.** *For every  $n \geq 1$ ,*

$$\min_{\substack{|X|=n \\ X \in \mathcal{A}_2}} D(X) = \min_{\substack{|X|=n \\ X \in \mathcal{A}_2}} M(X) = \left\lfloor \frac{n}{2} \right\rfloor,$$

and

$$\min_{\substack{|X|=n \\ X \in \mathcal{A}_2}} \Sigma(X) = n \left\lfloor \frac{n}{2} \right\rfloor.$$

*In particular, the global, pinned, and sum lower bounds at the  $n/2$  scale are all sharp.*

*Proof.* The lower bounds follow immediately from Lemma 2.1 with  $m = 2$ :

$$d_X(x) \geq \left\lceil \frac{n-1}{2} \right\rceil = \left\lfloor \frac{n}{2} \right\rfloor \quad (x \in X).$$

Hence

$$M(X) \geq \left\lfloor \frac{n}{2} \right\rfloor, \quad D(X) \geq \left\lfloor \frac{n}{2} \right\rfloor, \quad \Sigma(X) \geq n \left\lfloor \frac{n}{2} \right\rfloor.$$

It remains to prove sharpness. Let  $X$  be the vertex set of a regular  $n$ -gon, written as

$$X = \{\zeta^0, \zeta^1, \dots, \zeta^{n-1}\} \subset \mathbb{C} \cong \mathbb{R}^2, \quad \zeta = e^{2\pi i/n}.$$

Fix a vertex  $x_i = \zeta^i$ . For  $m \in \{1, \dots, n-1\}$ ,

$$|x_i - x_{i+m}| = |1 - \zeta^m| = 2 \sin\left(\frac{\pi m}{n}\right),$$

where indices are taken modulo  $n$ . Since

$$\sin\left(\frac{\pi m}{n}\right) = \sin\left(\frac{\pi(n-m)}{n}\right),$$

the distances from  $x_i$  occur in equal pairs corresponding to  $m$  and  $n - m$ , and these are the only repetitions because  $m \mapsto \sin(\pi m/n)$  is strictly increasing on  $1 \leq m \leq \lfloor n/2 \rfloor$ .

Therefore every circle centered at  $x_i$  meets at most two other vertices, so  $X \in \mathcal{A}_2$ . Moreover every vertex determines exactly

$$\left\lceil \frac{n-1}{2} \right\rceil = \left\lfloor \frac{n}{2} \right\rfloor$$

distinct distances, whence

$$M(X) = \left\lfloor \frac{n}{2} \right\rfloor, \quad \Sigma(X) = n \left\lfloor \frac{n}{2} \right\rfloor.$$

Finally, the global set of pairwise distances is exactly

$$\left\{ 2 \sin\left(\frac{\pi m}{n}\right) : 1 \leq m \leq \left\lfloor \frac{n}{2} \right\rfloor \right\},$$

so

$$D(X) = \left\lfloor \frac{n}{2} \right\rfloor.$$

This proves sharpness for all three quantities.  $\square$

**Corollary 3.2.** *Let  $\mathcal{F}_n$  be any family of  $n$ -point subsets of  $\mathbb{R}^2$  that contains the regular  $n$ -gon. Then*

$$\min_{X \in \mathcal{F}_n \cap \mathcal{A}_2} D(X) = \min_{X \in \mathcal{F}_n \cap \mathcal{A}_2} M(X) = \left\lfloor \frac{n}{2} \right\rfloor,$$

and

$$\min_{X \in \mathcal{F}_n \cap \mathcal{A}_2} \Sigma(X) = n \left\lfloor \frac{n}{2} \right\rfloor.$$

*Proof.* The lower bounds come from Theorem 3.1; the regular  $n$ -gon provides equality.  $\square$

*Remark 3.3.* Corollary 3.2 immediately applies to every family that still admits the regular  $n$ -gon, for instance:

- (i)  $\mathcal{N}_3 \cap \mathcal{A}_2$ ,
- (ii)  $\mathcal{C} \cap \mathcal{A}_2$ ,
- (iii) the family of cyclic  $n$ -gons satisfying  $\mathcal{A}_2$ .

Thus adding only “no three collinear” or only “convex position” to the literal statement of Problem #655 does *not* repair it: the exact sharp lower bound remains  $\lfloor n/2 \rfloor$ , and no improvement to  $(1+c)\frac{n}{2}$  is possible.

## 4 What the older Erdős sources actually ask

The modern website formulation mixes together several distinct strands from the older literature.

### 4.1 The 1987 general-position questions

In [10], Erdős begins with the global general-position problem: given  $n$  points in the plane in general position (no three on a line, no four on a circle), determine the minimum possible number of distinct pairwise distances. He writes that he has no example forcing this quantity to be  $o(n^2)$ , but also cannot prove it is superlinear.

In the same paper he then turns to the pinned quantity  $d_X(x)$ . Writing  $D(n) = \max_{x \in X} d_X(x)$  for a general-position set  $X$ , he notes the trivial bound  $D(n) \geq (n-1)/3$  and asks whether there is an absolute constant  $c > 0$  such that

$$D(n) > (1+c)\frac{n}{3}.$$

He also asks for a sum version

$$\sum_{x \in X} d_X(x) > (1+c)\frac{n^2}{3},$$

and explicitly remarks that the pinned  $n/3$  question might still be true under weaker assumptions, for instance under no-four-cocircularity, or even merely under the local condition that every circle centered at a point of  $X$  contains at most three other points of  $X$  [10]. In our notation, this is exactly the  $\mathcal{A}_3$  scale from Remark 2.2.

## 4.2 The 1988 strong local- $\mathcal{A}_2$ pinned question

In [11], Erdős asks a stronger question at the  $n/2$  scale. He considers  $n$  points  $x_1, \dots, x_n$  in the plane such that:

- (i) no four points are cocircular, and
- (ii) every circle centered at one of the  $x_i$  contains at most two other points.

He notes that then  $d_X(x_i) \geq n/2$  for every  $i$  and asks whether there exists an absolute constant  $c > 0$  with

$$\max_{x \in X} d_X(x) > (1+c)\frac{n}{2}.$$

He also suggests the sum version

$$\sum_{x \in X} d_X(x) > (1+c)\frac{n^2}{2},$$

and remarks that some cocircularity restriction is necessary because otherwise the regular polygon gives a counterexample. He further suggests trying to deduce this pinned  $n/2$  statement from as weak an assumption as possible [11].

This 1988 problem is the closest classical antecedent of the current webpage’s local- $\mathcal{A}_2$  formulation. The crucial difference is that Erdős asks for the *pinned* quantity  $M(X) = \max_x d_X(x)$ , whereas the current website statement asks only for the *global* quantity  $D(X)$ .

## 4.3 The website and the forum

The current webpage for Problem #655 asks for the global quantity  $D(X)$  under  $\mathcal{A}_2$ , records Hunter’s regular-polygon counterexample, and says that some general-position assumption was probably intended [4]. The discussion thread then raises two natural candidate repairs:

- (i) general position, and
- (ii) convex position,

and the database was updated to flag the statement as ambiguous [5].

Historically, both suggestions are reasonable:

- the general-position suggestion points directly back to [10];
- the convex-position suggestion points back to Erdős’ 1946 convex problem, Altman’s theorem, and the still-open pinned convex problem [9, 1, 2, 12].

The point of the present note is that these two repairs lead to *different* mathematics, and only one of them can plausibly repair the literal local- $\mathcal{A}_2$  statement.

# 5 Natural repairs and what is known about them

## 5.1 Adding only “no three collinear”

There are two distinct possibilities.

(a) **Keep the local condition  $\mathcal{A}_2$ .** Then nothing changes: by Remark 3.3, the regular  $n$ -gon still belongs to  $\mathcal{N}_3 \cap \mathcal{A}_2$ , so the exact sharp lower bounds are still

$$D(X), M(X) \geq \left\lfloor \frac{n}{2} \right\rfloor, \quad \Sigma(X) \geq n \left\lfloor \frac{n}{2} \right\rfloor,$$

and all are attained by the regular  $n$ -gon.

(b) **Drop  $\mathcal{A}_2$  and ask only for distinct distances among no-three-collinear sets.** This is a classical open problem. The upper bound  $\lfloor n/2 \rfloor$  still comes from the regular  $n$ -gon. The best general lower bound is the old Szemerédi argument, which we record for completeness.

**Proposition 5.1** (Szemerédi). *If  $X \subset \mathbb{R}^2$  has  $n$  points and no three points of  $X$  are collinear, then*

$$M(X) \geq \left\lceil \frac{n-1}{3} \right\rceil, \quad D(X) \geq \left\lceil \frac{n-1}{3} \right\rceil.$$

Hence, if

$$D_{\mathcal{N}_3}(n) := \min\{D(X) : |X| = n, X \in \mathcal{N}_3\},$$

then

$$\left\lceil \frac{n-1}{3} \right\rceil \leq D_{\mathcal{N}_3}(n) \leq \left\lfloor \frac{n}{2} \right\rfloor.$$

*Proof.* Let  $m := M(X)$ . For each  $a \in X$ , the  $n-1$  points of  $X \setminus \{a\}$  are partitioned into at most  $m$  classes according to their distance from  $a$ . If the class sizes are  $r_1, \dots, r_t$  with  $t \leq m$  and  $\sum r_i = n-1$ , then by Cauchy–Schwarz,

$$\sum_{i=1}^t r_i^2 \geq \frac{(n-1)^2}{m}.$$

Hence the number of unordered pairs  $\{p, q\} \subset X \setminus \{a\}$  with  $|a-p| = |a-q|$  is at least

$$\sum_{i=1}^t \binom{r_i}{2} = \frac{1}{2} \left( \sum_{i=1}^t r_i^2 - (n-1) \right) \geq \frac{(n-1)(n-1-m)}{2m}.$$

Summing over all centers  $a \in X$ , the total number  $T$  of unordered isosceles triples  $(a, \{p, q\})$  in  $X$  satisfies

$$T \geq n \cdot \frac{(n-1)(n-1-m)}{2m}.$$

On the other hand, for any unordered pair  $\{p, q\} \subset X$ , a point  $a \in X$  contributes an isosceles triple with base  $pq$  if and only if  $a$  lies on the perpendicular bisector of  $pq$ . Since  $X$  has no three collinear points, that bisector contains at most two points of  $X$ . Therefore

$$T \leq 2 \binom{n}{2} = n(n-1).$$

Combining the two bounds and cancelling  $n(n-1)$  gives

$$\frac{n-1-m}{2m} \leq 1, \quad \text{so} \quad n-1 \leq 3m.$$

Thus  $m \geq (n-1)/3$ , and therefore  $M(X) \geq \lceil (n-1)/3 \rceil$ . Since  $D(X) \geq M(X)$ , the same lower bound holds for  $D(X)$ .  $\square$

Erdős later recorded that Szemerédi conjectured the upper bound to be sharp, i.e. that every no-three-collinear  $n$ -point set determines at least  $\lfloor n/2 \rfloor$  distances, while only the  $n/3$  lower bound is known [12]. Thus the no-three-collinear repair of the literal website problem does not become exact or even close to exact: it becomes a different classical open problem.

## 5.2 Convex position

Again there are two possibilities.

**(a) Keep the local condition  $\mathcal{A}_2$ .** Then, by Corollary 3.2, nothing changes: the regular  $n$ -gon belongs to  $\mathcal{C} \cap \mathcal{A}_2$ , so the exact minima remain

$$D(X) = M(X) = \left\lfloor \frac{n}{2} \right\rfloor, \quad \Sigma(X) = n \left\lfloor \frac{n}{2} \right\rfloor.$$

Hence convexity alone does *not* rescue the literal webpage statement.

**(b) Drop  $\mathcal{A}_2$  and ask classical convex-position questions.** Then the global and pinned problems diverge sharply.

For the global quantity,

$$D_{\mathcal{C}}(n) := \min\{D(X) : |X| = n, X \in \mathcal{C}\},$$

Erdős conjectured in 1946 that  $D_{\mathcal{C}}(n) = \lfloor n/2 \rfloor$ ; Altman proved this, and the regular  $n$ -gon shows sharpness [9, 1, 2]. So the convex *global* version is already solved.

For the pinned quantity,

$$M_{\mathcal{C}}(n) := \min\{M(X) : |X| = n, X \in \mathcal{C}\},$$

the exact value is still unknown. Erdős conjectured that one always has  $M_{\mathcal{C}}(n) = \lfloor n/2 \rfloor$ . Moser proved the lower bound  $M_{\mathcal{C}}(n) \geq \lceil n/3 \rceil$ , Dumitrescu improved this to

$$M_{\mathcal{C}}(n) \geq \left\lceil \frac{13n - 6}{36} \right\rceil,$$

and Nivasch, Pach, Pinchasi, and Zerbib further improved it to

$$M_{\mathcal{C}}(n) \geq \left( \frac{13}{36} + \frac{1}{22701} \right) n + O(1),$$

while the regular  $n$ -gon still gives the upper bound  $M_{\mathcal{C}}(n) \leq \lfloor n/2 \rfloor$  [15, 6, 16, 17].

Thus Bloom's forum comment that "convex position seems one natural interpretation" is historically accurate, but the mathematically natural convex interpretation is the *pinned* problem, not the literal local- $\mathcal{A}_2$  global one.

## 5.3 General position and no-four-cocircularity

Here one finally excludes the regular-polygon counterexample mechanism. There are again several non-equivalent questions.

**(a) General-position global distances.** Define

$$D_{\mathcal{G}}(n) := \min\{D(X) : |X| = n, X \in \mathcal{G}\}.$$

This is the classical general-position distinct-distances problem already asked in [10] and revisited in [11]. By Remark 2.2, one has the trivial linear lower bound

$$D_{\mathcal{G}}(n) \geq \left\lceil \frac{n-1}{3} \right\rceil.$$

The best known upper bound is the construction of Erdős, Füredi, Pach, and Ruzsa,

$$D_{\mathcal{G}}(n) \leq n \exp(C\sqrt{\log n})$$

for some absolute constant  $C > 0$  [8]; see also Dumitrescu’s discussion [7] and Sheffer’s survey [17]. It remains open whether  $D_{\mathcal{G}}(n)/n \rightarrow \infty$ , or even what the correct order of magnitude is.

**(b) General-position pinned distances.** Define

$$M_{\mathcal{G}}(n) := \min\{M(X) : |X| = n, X \in \mathcal{G}\}.$$

Again Remark 2.2 gives the trivial lower bound

$$M_{\mathcal{G}}(n) \geq \left\lceil \frac{n-1}{3} \right\rceil.$$

The 1987 source asks whether one can improve this to  $(1+c)\frac{n}{3}$  for some absolute  $c > 0$  [10], and the modern page for Problem #654 records the same question, together with the more optimistic possibility that perhaps  $M_{\mathcal{G}}(n) > (1-o(1))n$  under stronger local assumptions [3]. This pinned general-position problem remains open.

**(c) The closest repaired version of the literal website statement.** If one wants a repair that stays at the original  $n/2$  scale and keeps the local condition  $\mathcal{A}_2$ , the most faithful version is not a global statement about  $D(X)$  but the 1988 pinned question:

If  $X$  has no four cocircular points and satisfies  $\mathcal{A}_2$ , must some point of  $X$  determine more than  $(1+c)\frac{n}{2}$  distinct distances?

Formally, if one defines

$$M_{\mathcal{N}_4 \cap \mathcal{A}_2}(n) := \min\{M(X) : |X| = n, X \in \mathcal{N}_4 \cap \mathcal{A}_2\},$$

then Lemma 2.1 gives the trivial lower bound

$$M_{\mathcal{N}_4 \cap \mathcal{A}_2}(n) \geq \left\lfloor \frac{n}{2} \right\rfloor,$$

and Erdős asks whether this can be improved to  $(1+c)\frac{n}{2}$  [11]. Since  $D(X) \geq M(X)$  for every  $X$ , any positive answer to this pinned question would immediately imply the analogous repaired *global* statement

$$D(X) > (1+c)\frac{n}{2}$$

under the same hypotheses.

In this precise sense, the most natural repaired version of the literal webpage statement is the pinned no-four-cocircular-plus- $\mathcal{A}_2$  problem of [11].

*Remark 5.2.* The current webpage for Problem #654 additionally records, as of 2026, that the strongest no-four-cocircular pinned conjecture without the stronger general-position hypothesis has a counterexample with  $M(X) \leq 3n/4$  for every point, using a set lying on two lines [3]. We record this only as a database remark, since we have not traced that construction to a primary published source.

## 5.4 Sum versions

The same trichotomy appears for the sum

$$\Sigma(X) = \sum_{x \in X} d_X(x).$$

Under  $\mathcal{A}_2$ , Theorem 3.1 gives the exact sharp minimum

$$\Sigma(X) \geq n \left\lfloor \frac{n}{2} \right\rfloor,$$

with equality for the regular  $n$ -gon.

At the  $n/3$  scale, Erdős asked in 1987 whether general-position sets satisfy

$$\Sigma(X) > (1 + c) \frac{n^2}{3}$$

for some absolute  $c > 0$  [10]. At the  $n/2$  scale, he suggested in 1988 the stronger local- $\mathcal{A}_2$  no-four-cocircular version

$$\Sigma(X) > (1 + c) \frac{n^2}{2}$$

[11]. As far as the sources we checked indicate, these sum problems remain open.

## 6 A summary chart

Table 1 summarizes the variants most relevant to the forum discussion.

Assumptions	Quantity	Status	Best general statement
$\mathcal{A}_2$	$D(X), M(X)$	exact	$\min = \lfloor n/2 \rfloor$ (Theorem 3.1); regular $n$ -gon extremal.
$\mathcal{A}_2$	$\Sigma(X)$	exact	$\min = n \lfloor n/2 \rfloor$ (Theorem 3.1); regular $n$ -gon extremal.
$\mathcal{N}_3 \cap \mathcal{A}_2$	$D(X), M(X), \Sigma(X)$	exact	Same exact bounds as for $\mathcal{A}_2$ ; regular $n$ -gon still admissible.
$\mathcal{C} \cap \mathcal{A}_2$	$D(X), M(X), \Sigma(X)$	exact	Same exact bounds as for $\mathcal{A}_2$ ; regular $n$ -gon still admissible.
$\mathcal{N}_3$	$D(X)$	open	$\lceil (n-1)/3 \rceil \leq D_{\mathcal{N}_3}(n) \leq \lfloor n/2 \rfloor$ (Proposition 5.1); conjecturally the upper bound is sharp.
$\mathcal{C}$	$D(X)$	exact	$D_{\mathcal{C}}(n) = \lfloor n/2 \rfloor$ (Altman).
$\mathcal{C}$	$M(X)$	open	$(\frac{13}{36} + \frac{1}{22701})n + O(1) \leq M_{\mathcal{C}}(n) \leq \lfloor n/2 \rfloor$ ; exact value unknown.
$\mathcal{G}$	$D(X)$	open	$\lceil (n-1)/3 \rceil \leq D_{\mathcal{G}}(n) \leq n \exp(C\sqrt{\log n})$ for some $C > 0$ .
$\mathcal{G}$	$M(X)$	open	$M_{\mathcal{G}}(n) \geq \lceil (n-1)/3 \rceil$ ; ask whether $> (1+c)n/3$ , perhaps even $(1-o(1))n$ under stronger local hypotheses.
$\mathcal{N}_4 \cap \mathcal{A}_2$	$M(X)$	open	$M(X) \geq \lfloor n/2 \rfloor$ trivially; Erdős asks whether $> (1+c)n/2$ . This is the most faithful open repair of the literal #655 statement.

Table 1: Variants nearest to the current formulation of Erdős Problem #655.

## 7 Conclusion

The forum’s two suggested repairs—general position and convex position—both have real historical precedent, but they lead to different problems.

- If one keeps the local condition  $\mathcal{A}_2$  and adds only assumptions that still allow the regular  $n$ -gon (for example no three collinear or convex position), then nothing changes: the exact sharp lower bound is still  $\lfloor n/2 \rfloor$  for both the global and pinned quantities, and  $n\lfloor n/2 \rfloor$  for the sum.
- If one drops  $\mathcal{A}_2$  and asks for global distinct distances in convex position, one recovers a classical solved problem of Altman.
- If one drops  $\mathcal{A}_2$  and asks for global or pinned distinct distances in general position, one recovers the classical open  $n/3$ -scale problems already present in Erdős’ 1987 paper.
- If one wants a repaired statement nearest in spirit to the current webpage—namely, a statement that keeps the  $n/2$  scale and keeps the local “centered circles” hypothesis—then the right version is the pinned no-four-cocircular-plus- $\mathcal{A}_2$  problem from Erdős’ 1988 paper.

So the regular polygon does more than disprove the literal statement of Problem #655: it cleanly separates the variants that are already exactly settled from the variants that remain genuinely interesting.

## References

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